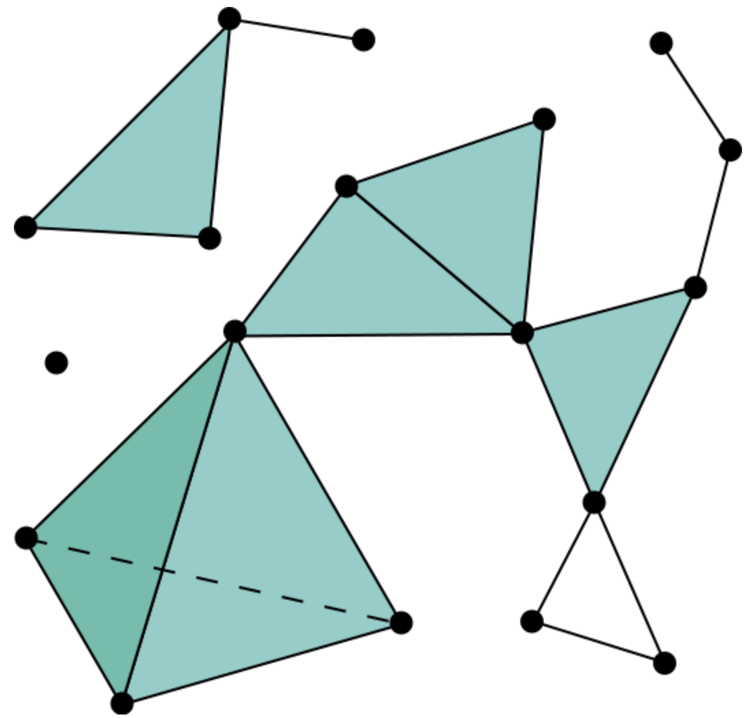
Investigating the Vietoris-Rips Complex

Searching for links in data sets

Simplicial Complexes

A Geometric Approach

- A set of Interconnected *Simplices*:
  - Points
  - Lines
  - Triangles
  - Tetrahedron
  - Pentachoron
  - Etc.



Simplicial Complexes (continued)

A Combinatorial Approach

Let; $A = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, ..., \vec{v}_{n+1}\} \subset \mathbb{R}^{n+1}$; a basis of $\mathbb{R}^{n+1}$.

Let S_A be the smallest convex set s.t. $A \subset S_A \subset \mathbb{R}^{n+1}$.

We call this new set an *n-simplex*

Simplicial Complexes (continued)

Let $X \subset \mathbb{R}^n$ be a data set:

$$X = \{x_1, x_2, x_3, ..., x_m\}$$

Define 2^X to be the set of all subsets of X.

We call $\bar{X}$ a simplicial complex on X if $\bar{X} \subset 2^X$.
(A subset of the set of all subsets of X.)

Note: our simplicial complex must also fullfill the consistency condition:
If A is an element of a simplicial complex then every subset of A is also in the simplicial complex. (this makes sure that we include the boundaries of every simplex)

Constructing the Vietoris-Rips Complex

Consider a specific Simplicial complex ; $\bar{X}_\epsilon$.

This set is defined by: for all $Y \in \bar{X}_\epsilon$, for all $x_i, x_k \in Y$, we have that $d(x_i, x_k) < \epsilon$.

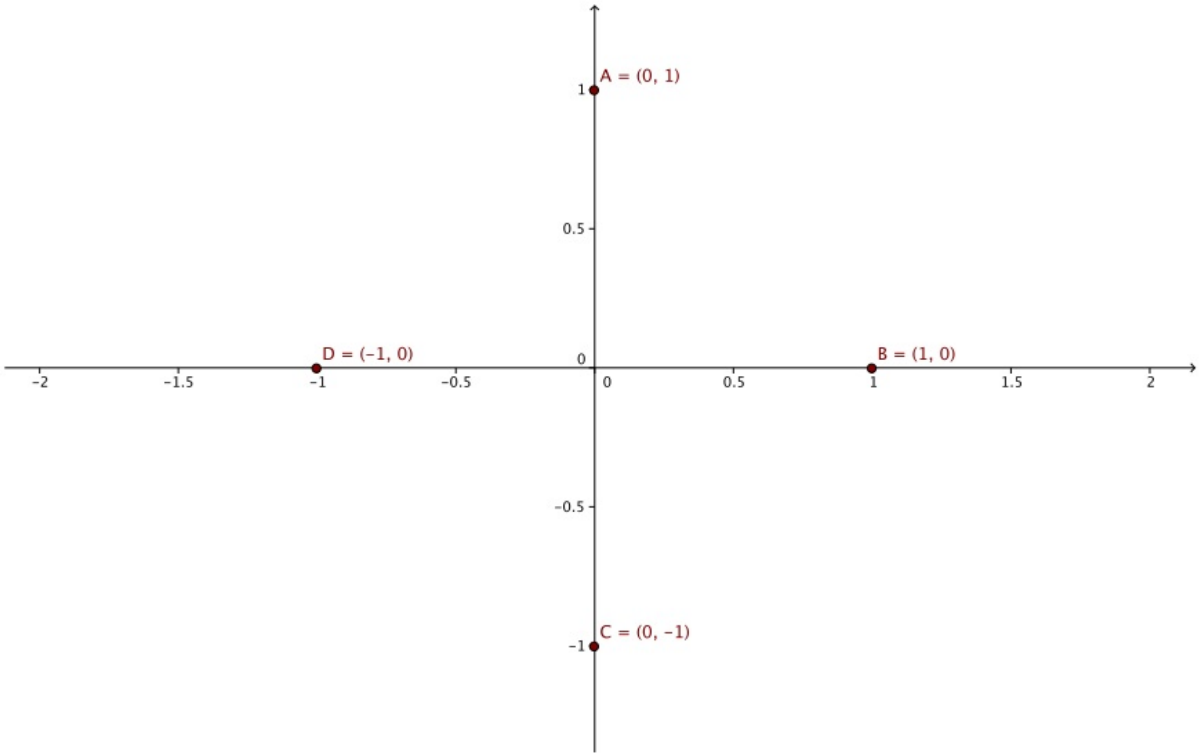
(we have bounded the pairwise length between any two points in the simplices that make up the simplicial complex)

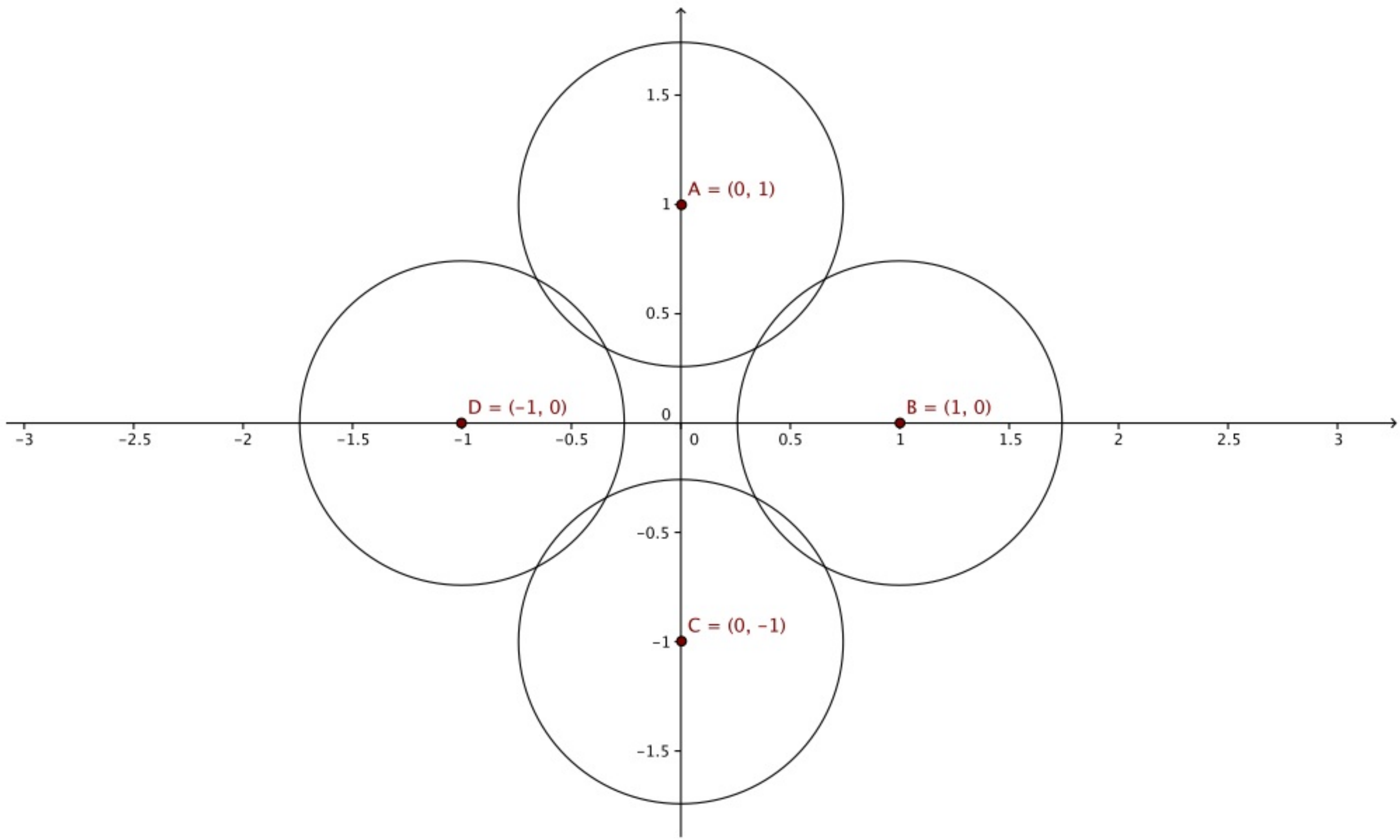
We call $\bar{X}_\epsilon$ the Vietoris-Rips Complex for the specific radius ϵ.

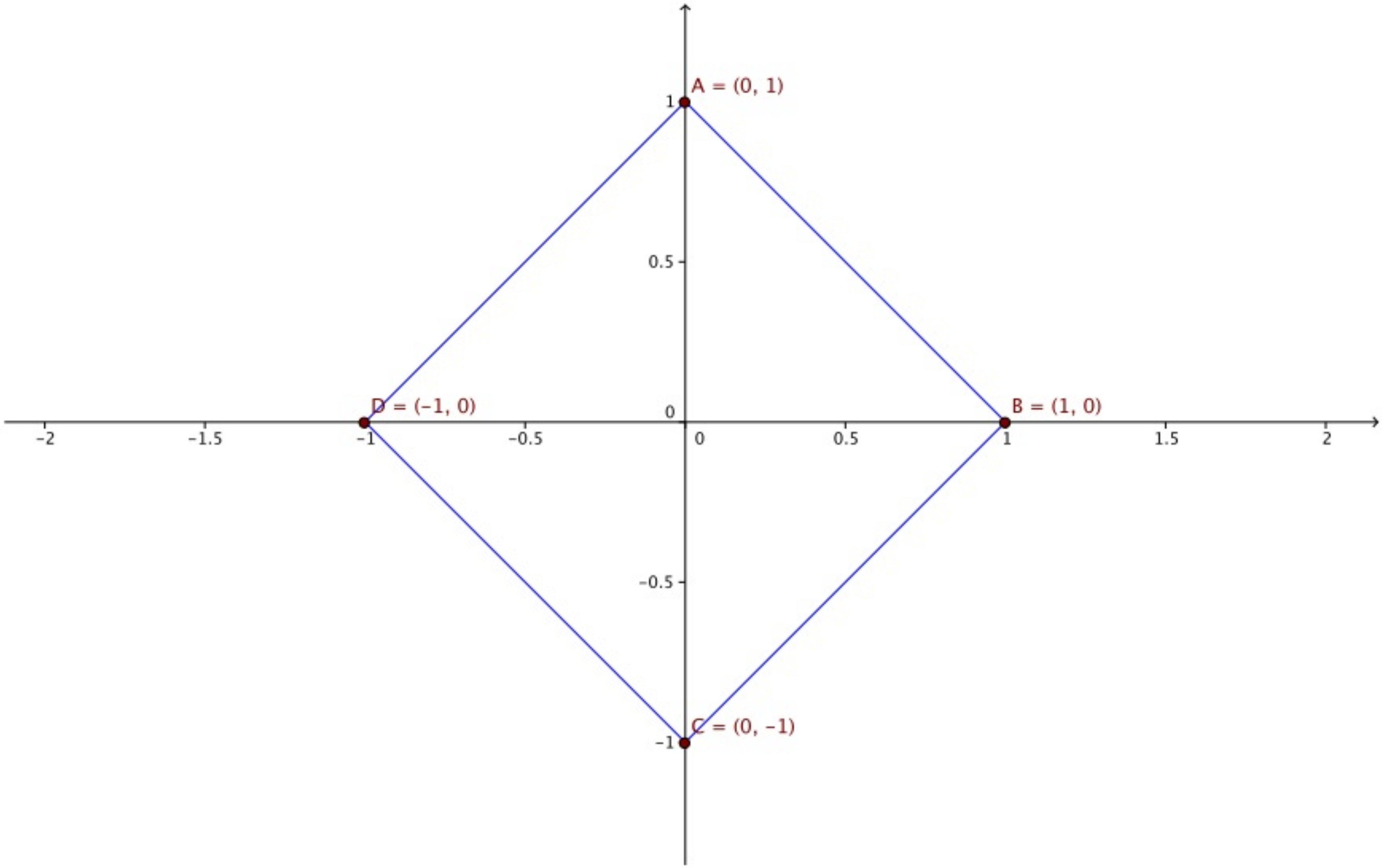
Constructing the Vietoris-Rips Complex

- Consider the following set of points:

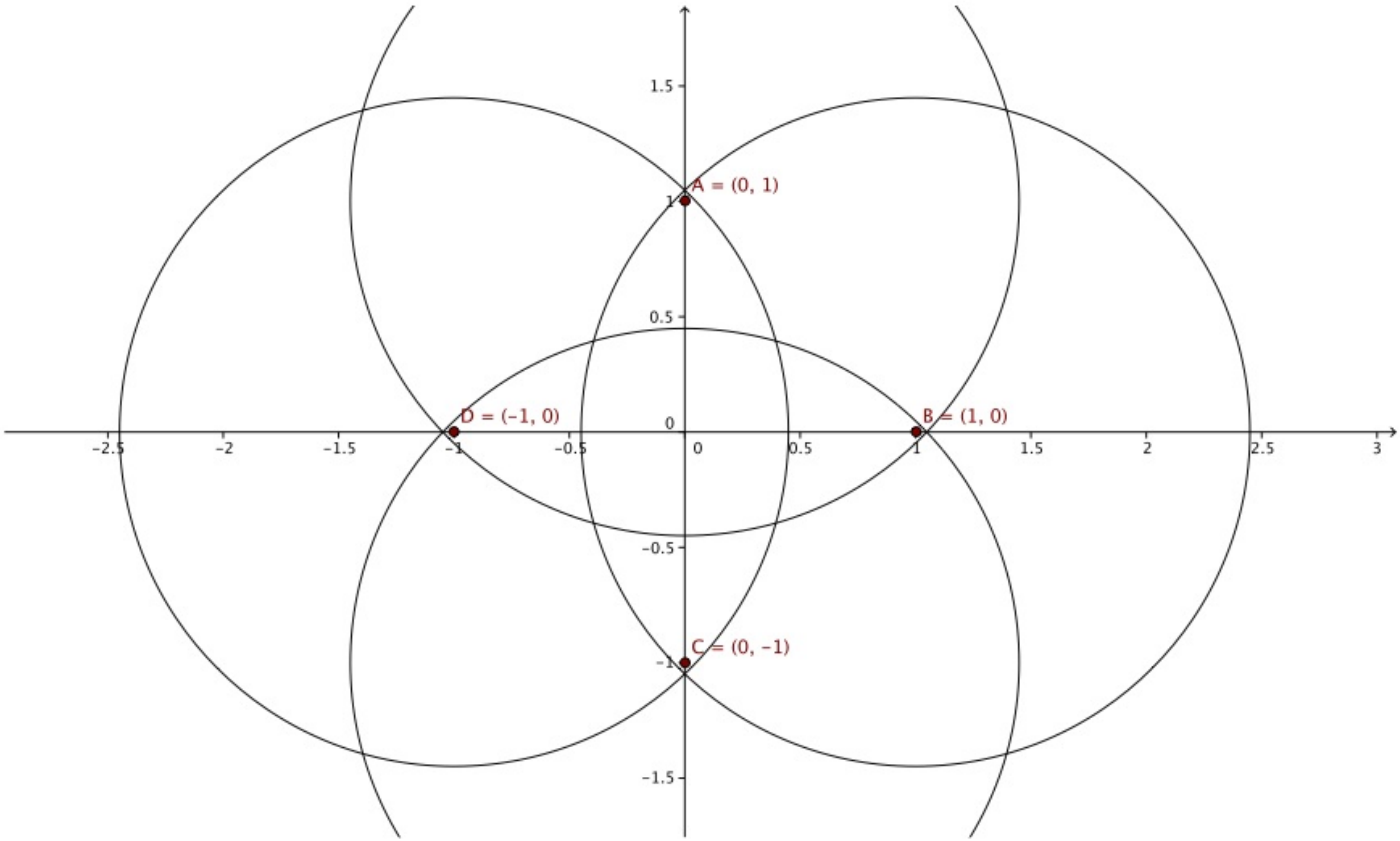
  A={(0,1), (1,0),(0,-1), (-1,0)}



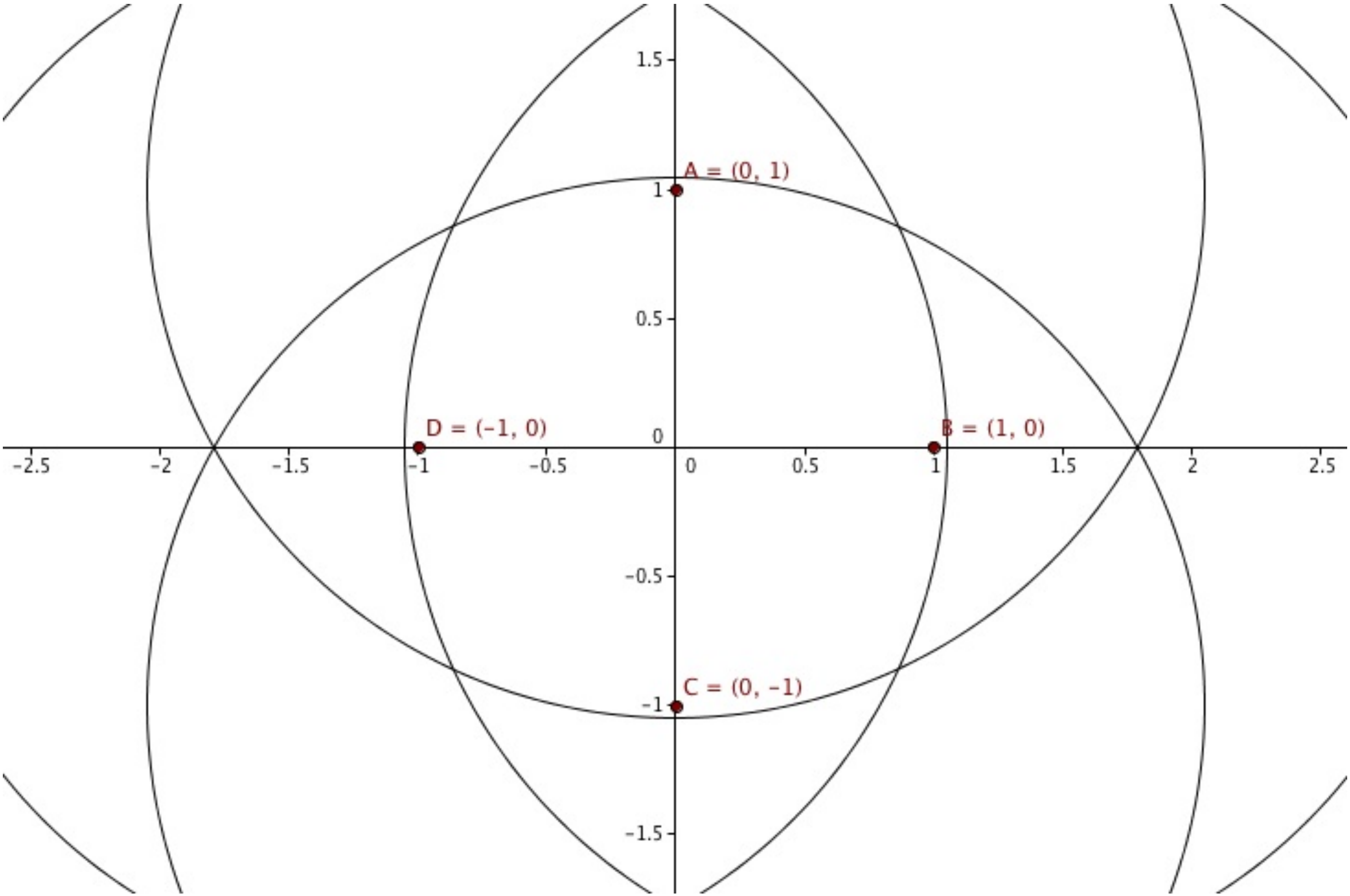

A = (0, 1)
B = (1, 0)
C = (0, −1)
D = (−1, 0)
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
1.5
1
0.5
0
−0.5
−1
−1.5



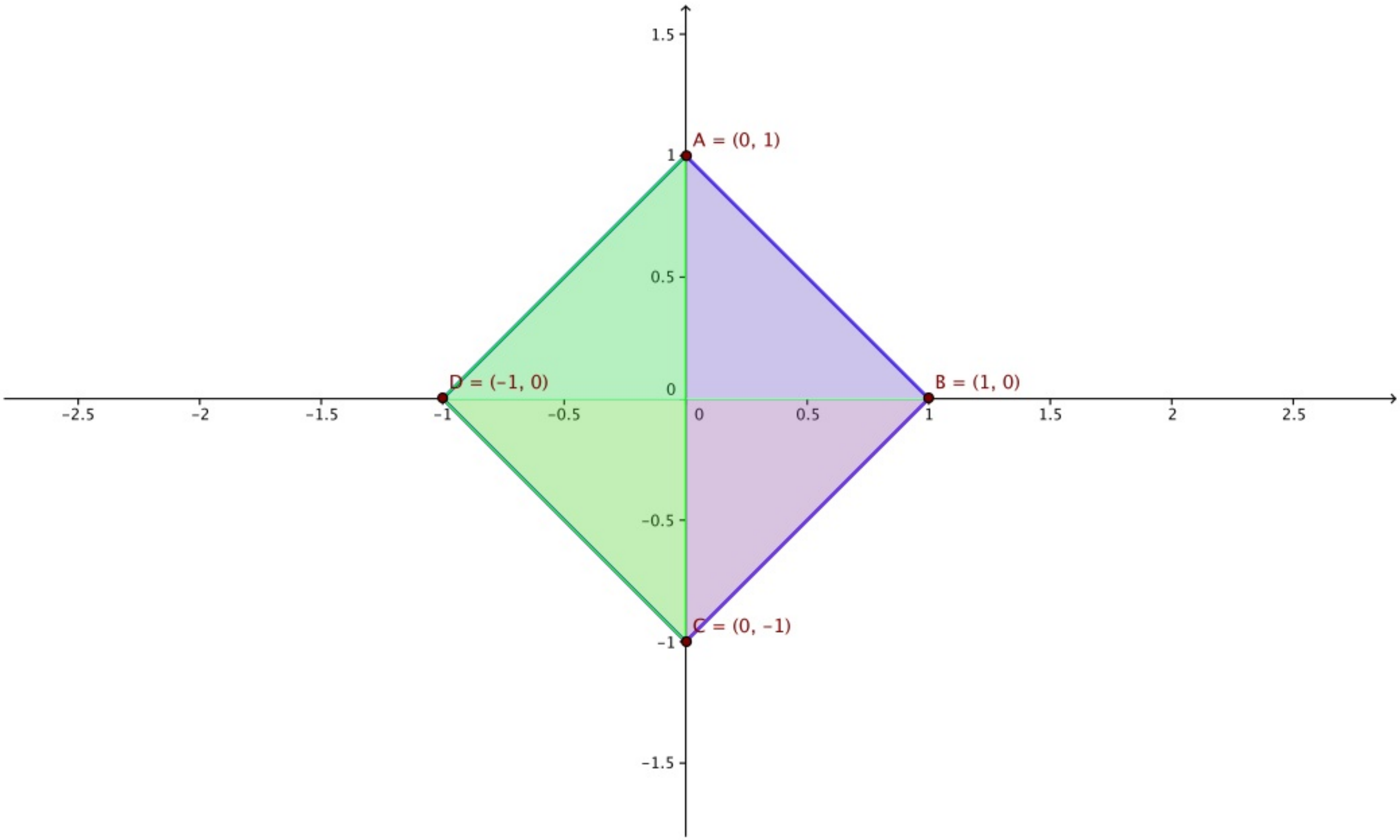
A = (0, 1)
B = (1, 0)
C = (0, −1)
D = (−1, 0)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
0.5
0
-0.5
-1



A = (0, 1)
B = (1, 0)
C = (0, −1)
D = (−1, 0)
1.5
1
0.5
0
−0.5
−1
−1.5
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5
3

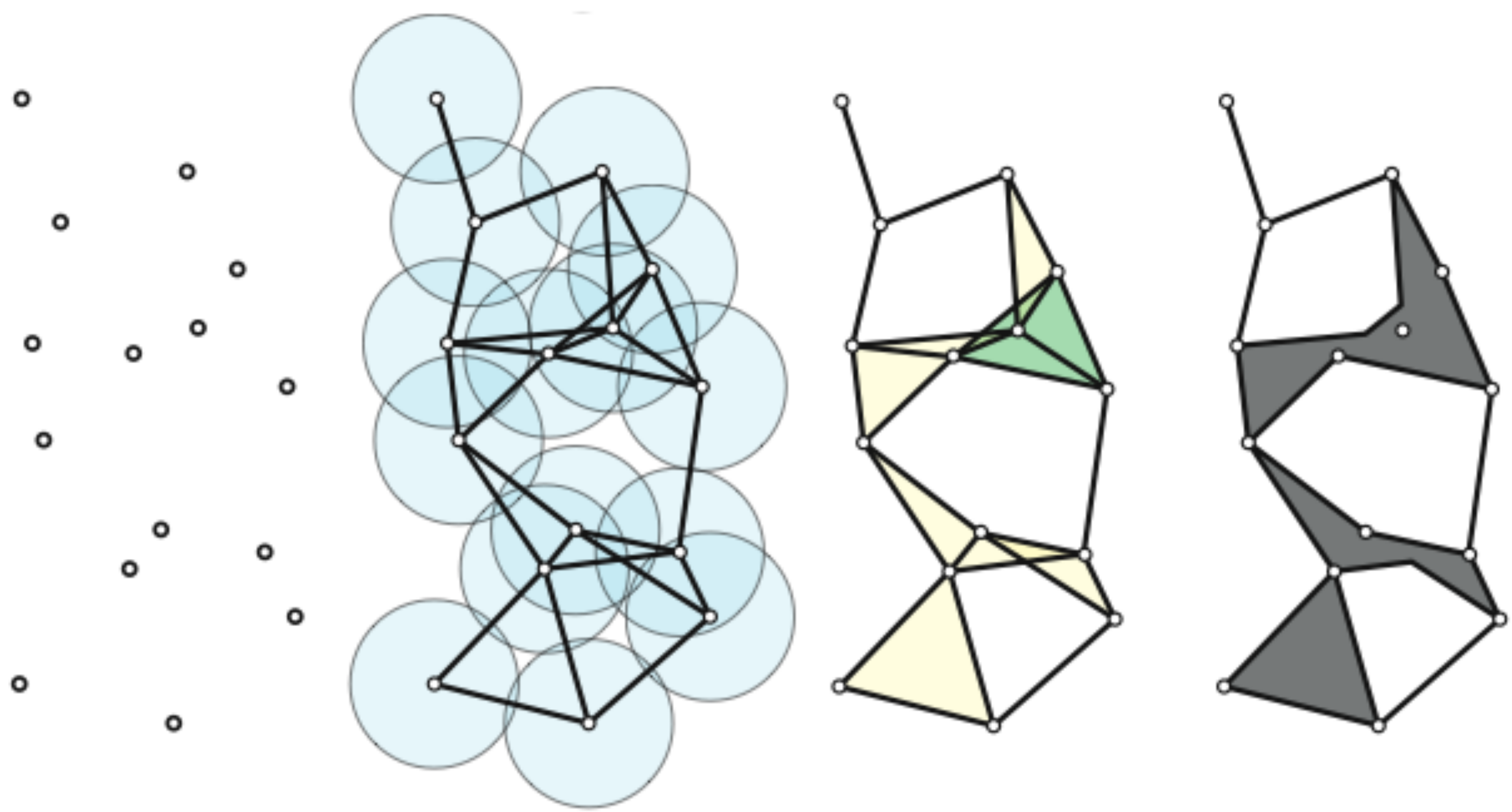


A = (0, 1)
B = (1, 0)
C = (0, −1)
D = (−1, 0)
1.5
1
0.5
0
−0.5
−1
−1.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5



Displayed Simplices:
[{(0,1)}; {(1,0)}; {(0,-1)}; {(-1,0)};
{(0,1),(1,0)}; {(1,0),(0,-1)}; {(0,-1),(-1,0)}; {(-1,0),(0,1)};
{(0,1),(1,0),(0,-1)}; {(1,0),(0,-1),(-1,0)}; {(0,-1),(-1,0),(0,1)}; {(-1,0),(0,1),(1,0)};
{(0,1),(1,0),(0,-1),(-1,0)}]

A Better Picture:

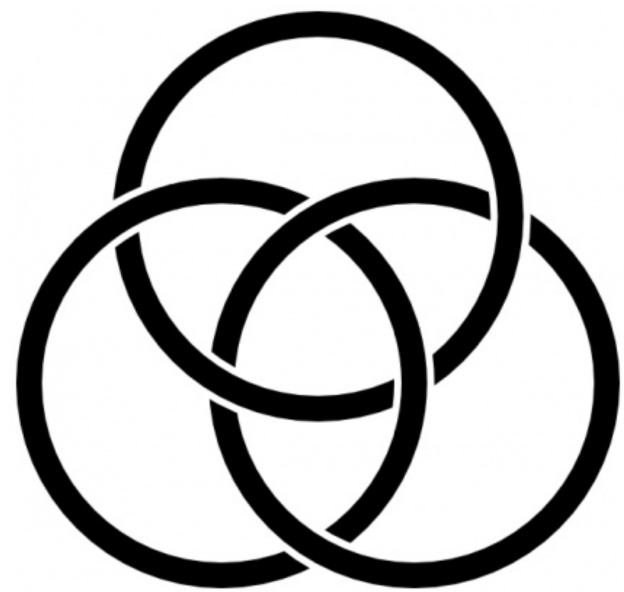


Cycles

- What we are looking for are cycles.
- Can be thought of as holes in the Vietoris-Rips Complex.
- We are interested in these holes because they are topologically distinct from just points.

- More specifically, we are searching for methods of detecting links; non-intersecting, but interlocked cycles.

- Borromean Rings:



Using Perseus

- One method being investigated is using the program “Perseus”
  - Perseus is a program that accepts a data set and returns descriptions of the generated cycles and overall number of simplexes as the value of epsilon is increased for the given Vietoris-Rips Complex.
  - Possible to track the life of different cycles until they are swallowed by increasingly loose distance requirements. (Increasing value of epsilon)
- Discretization of Convenient Functions
  - We can take already conveniently constructed functions displaying certain qualities (like a link) and choose a discrete subset of the functions to test if Perseus can determine the behavior of the “data”, or if a tweaked code can even detect the links.
- Possibility of applying to real life data to determine behavior.

References:

- Vietoris–Rips Complexes of Planar Point Sets

  Erin W. Chambers, Vin de Silva, Jeff Erickson and Robert Ghrist

  Discrete & Computational Geometry, 2010, Volume 44, Number 1, Pages 75-90

- Borromean Rings: Mohamed Ibrahim www.clker.com
- Perseus Program: Vidit Nanda
- Robet Ghrist